To start the process, the value of the EMA is set to the n-period simple average of the prior n values of data. Some programs simply assume that the first value of x is approximately equal to the average. This formula is called an exponential average, because it can be written as a series expansion of the form:

$$EMA_n(t) = k_n \sum_{i=0}^{\infty} (1 - k_n)^i x_{t-i}$$

This formulation highlights one of the problems with starting up an EMA that can be very troublesome with automated systems. When we start the EMA, we are truncating this exponential series. Table 4 below shows the impact of this. There are three solutions to the truncation problem. The first is simply to use substantial quantities of data. This works well if we have an essentially infinite supply of data without gaps for both training and execution of the model. The second is to avoid exponential moving averages, and use a simple moving average instead. The third is to compute an **exponentially weighted moving average** (EWMA) which eliminates the error. This approach can be used to replace the EMA itself as well as when applying an EMA smoothing an indicator.

Insert TABLE 4 here

Table 4 - The maximum error in computing an n-period EMA when the series is truncated at different points. Each row represents a different number (p) of examples over which the EMA was computed. Each column represents a different smoothing period (n). Typical errors are less than those in the table.

The n-period EWMA over *p* data examples is computed using the following formula. Notice that this formula requires the prior *p* values of the data for each computation. If you are writing your own technical indicators, this corrected or EWMA will often improve the performance and consistency of your system. At present, the author is not aware of any commercially available systems that implement this.

$$EWMA_{n,p}(t) = \frac{\sum_{i=0}^{p-1} (1-k_n)^i x_{t-i}}{\sum_{i=0}^{p-1} (1-k_n)^i}$$

Where: *n* is the smoothing period

p   is the window over which the exponential average is computed.

k_n = 1/(n+1) is the smoothing coefficient.

x_t is the value of the time series at time t.

EWMA $_{n,p}$ (t) is the n-period EWMA evaluated at time t over p data points.